Individual Bunch Longitudinal Instabilities

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In this note we take a look at the question of longitudinal stability for single, individual bunches in the Energy Doubler (ED).

1. Definition of Longitudinal Coupling Impedance

Consider a circular machine (ED) with average radius R and denote with θ the angular coordinate (angle) along the main closed orbit with the same radius R. This angle increases by 2π every revolution around the machine.

Let λ and I be respectively the charge per unit length and the current associated to a charged beam circulating in the accelerator. Because of the angular periodicity, with a Fourier expansion we have

$$\lambda(\theta,t) = \sum_n \lambda_n(t)\, e^{in(\theta - \omega_0 t)} \tag{1a}$$

and

$$I(\theta,t) = \sum_n I_n(t)\, e^{in(\theta - \omega_0 t)} \tag{1b}$$

where ω_0 is the angular (reference) revolution frequency. Eqs. (1a) and (1b) are quite general in the sense they apply to any sort of beam, bunched and unbunched. Also, quite generally, the Fourier amplitudes λ_n and I_n can be varying with the time t. This variation would describe a perturbation within the beam, nevertheless since in any practical case, this variation is slowly compared to the revolution period, they can be regarded simply as an amplitude time-modulation.

Because of the law of conservation of charge

$$I_n = \beta c \,\lambda_n$$

βc being the velocity of the beam and c that of the light.

The longitudinal force acting to the center of the beam is simply given by the longitudinal component of the electric field produced by (1a) and (1b)

$$E_z(\theta, t) = \sum_n E_n(t) \, e^{in(\theta - \omega_0 t)}$$

In the approximation[1-5]

$$|n| \ll \gamma R/b \qquad , \qquad \gamma = 1/\sqrt{1 - \beta^2} \qquad (2)$$

we have

$$E_n = -in \, C_{bw} \frac{\lambda_n}{R} - \frac{Z_n I_n}{2\pi R} \qquad (3)$$

The first term is usually called the Beam Self-Field. The quantity C_{bw} is the remaining capacitance per unit length between beam and surrounding wall after the magnetic cancellation. For circular geometry where the vacuum pipe has radius b and the beam radius a

$$C_{bw} = (1 - \beta^2) \left(1 + 2 \ln \frac{b}{a}\right)$$

This quantity depends very drastically on the shape of the vacuum chamber.

The second term at the r.h.s. of (3) is usually called the Beam Image Field, and for large energies (γ) is usually the predominant one. The quantity Z_n is the total impedance of the media surrounding the beam at the vacuum chamber radius. It depends only on the electro-magnetic properties of the wall and not on the vacuum chamber cut-off. Quite often Z_n is referred to as the Longitudinal Coupling Impedance.

In the following, when we make applications to the Energy Doubler, we shall not consider the space-charge term,

namely the first term at the r.h.s. of (3). Also, in the case
of the Energy Doubler (R = 1000 m, b = 4 cm) at 100 GeV, the
limit (2) covers a very comfortable frequency range up to 500 GHz.

2. Phenomenological Description of Z_n

The impedance Z_n is obviously a complex quantity. Its
real part will be referred to as a Resistance and the imaginary
part as a Reactance. Nevertheless the notation we are using
here (see Eqs.(1a and b)) is such that an Inductance is a
negative reactance and a Capacitance a positive one.

The impedance Z_n is the result of the contribution of
all the items that make up the vacuum chamber wall and that
surround the beam all around the accelerator.

A particular impedance model which is now quite commonly
used in accelerator physics[5-13] is the one shown in Fig. 1.
A machine is made of many elements. Basically, all of them are
inductive in the low frequency range. They resonante, more or
less sharply at various frequencies. The medium frequency range
is reached when all the elements are practically past their
resonances. Past this range all the elements have a capacitive
behavior. There are three features of this model:

    a. Overall the impedance of the machine looks like that
       of a broad-band, low-Q resonator with a resonance
       frequency of one or few GHz and a shunt impedance Z_R.

    b. In the low frequency range there is an overall
       linear increase of Z_n with the frequency; in this
       range $|Z_n/n|$ is about constant.

    c. In the same low-frequency range, nevertheless one
       can recognize several but isolated sharp resonances,
       to which one can associate a high Q value and a large

shunt impedance.

It is well known that a sharp resonance impedance gives
rise to a long range wake field,
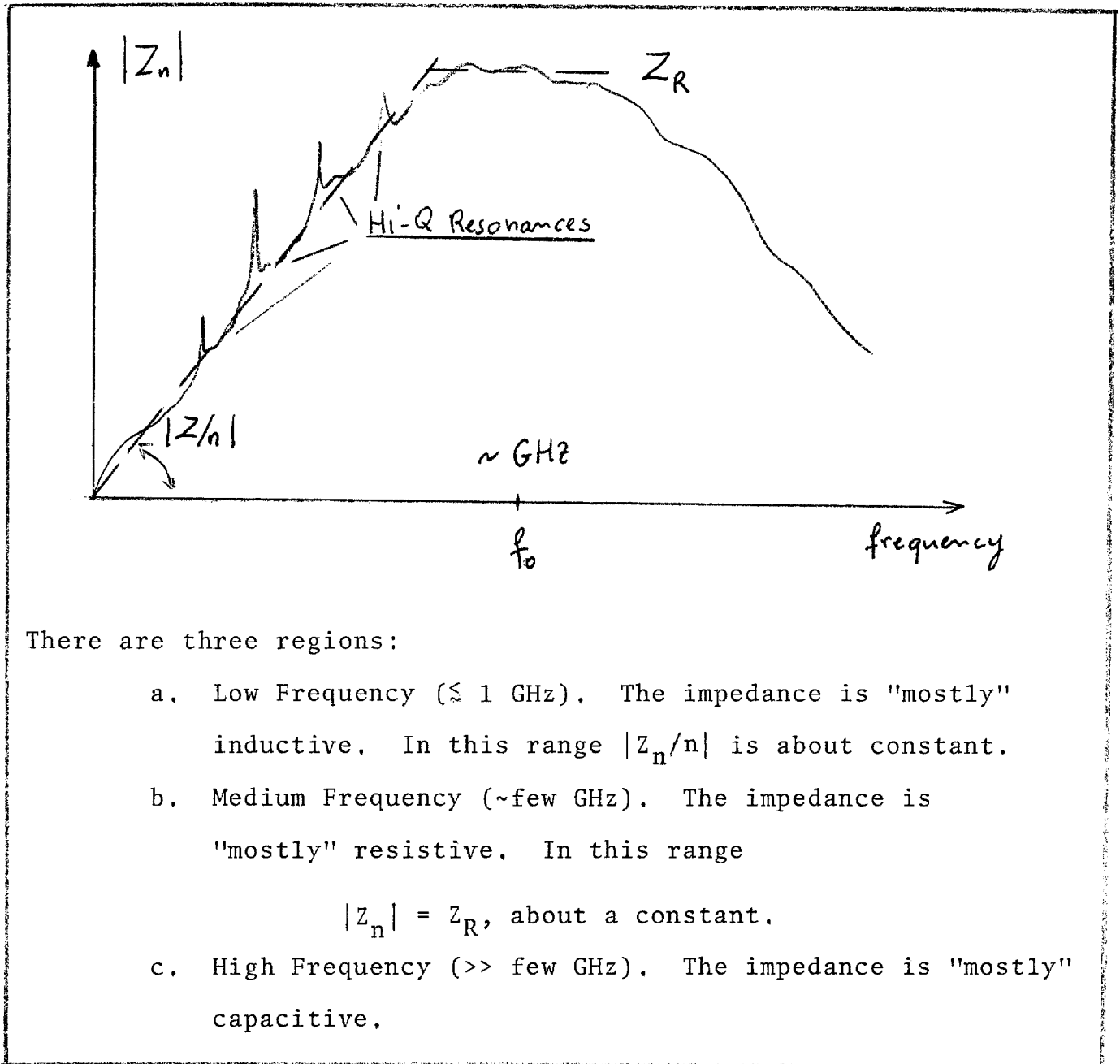
There are three regions:

    a. Low Frequency ($\lesssim$ 1 GHz). The impedance is "mostly"
       inductive. In this range $|Z_n/n|$ is about constant.

    b. Medium Frequency (~few GHz). The impedance is
       "mostly" resistive. In this range

$$|Z_n| = Z_R, \text{ about a constant.}$$

    c. High Frequency (>> few GHz). The impedance is "mostly"
       capacitive.

Figure 1. Phenomenological description of Z_n.

when excited by a beam bunch and therefore can cause a bunch-
to-bunch instability. Whereas a very low-Q impedance can give
rise only to a short range wake field and initiate an instability

among the particles of the same bunch. Since in this note we shall consider only individual bunch instabilities we will not consider here the contribution of sharp resonances.

Thus a machine is described by three parameters:

- the slope $|Z/n|$

- the resistance Z_R

- the frequency f_o at which $|Z_n|$ has a maximum. Observe that these parameters are also enough to define a low-Q resonator, a model that has been pursued recently.[8-9]

3. Estimation of the Impedance

The impedance (Z/n and Z_R) can be obtained by summing all the contributions of the various items which make the vacuum chamber. The contribution of each of these items can

(i)   be measured on a bunch[14-17]

(ii)   calculated according to some theoretical model.

For instance:

a.   Wall of homogeneous and isotropic material[2,5]

$$Z_n = \frac{\alpha\,\zeta}{b\,c}\,(2\pi R) \qquad (4)$$

ζ, is the characteristic surface impedance of the material.

b.   A special case of (4) when

$$\zeta = (1-i)\sqrt{\frac{n\,\omega_o\,\mu}{8\pi\sigma}}$$

which corresponds to a "resistive wall"[18]. μ is the magnetic permeability (relative), σ is the conductivity of the material. For the Energy Doubler we take

$$\mu = 1$$

$$\rho = 1/\sigma = 52 \; \mu\Omega \times cm \text{ for stainless}$$
$$\text{steel at } 4.2^{\circ} \text{ K}$$

and one has

$$Z_n = (1-i) \; 8.1 \; \sqrt{n} \text{ ohm.}$$

c. Bellows[19] (low frequency range)

$$Z_n = -i \; 4\pi \; \frac{n}{c} \; \beta \; \frac{\tau}{b} \; \frac{M\ell}{2\pi R}$$

M, total number of bellows

ℓ, length

τ, outer radius minus inner radius.

For the Energy Doubler, assuming unshielded
bellows, namely with no RF fingers and without
the vacuum chamber extension

$\tau = 0.635 \text{ cm}$

$M \sim 1000$

and

$|Z_n/n| = 0.27 \text{ ohm.}$

d. Conductive plates[20,21] (Pick-ups, Clearing
Electrodes, etc.)

$$Z_n = -2\pi i n \; Z_o \; \frac{M\ell}{2\pi R} \left(\frac{\phi_o}{\pi}\right)^2 \; P_n$$

where

ℓ is the length of the plates

ϕ_o, their semi-angular aperture

M, their number

Z_o, the matching termination

and $P_n = 1$ in the limit of wavelengths longer than the plates length.

For the ED

$\ell = 12$ cm

$\phi_0/\pi \sim \frac{1}{2}$

$M \sim 250$

$Z_0 = 50$ ohm

and

$|Z_n/n| = 0.4$ ohm

e. Vacuum Chamber Step[22,23]

In the low frequency range

$$\left| Z/n \right| = Z_0 \frac{(S-1)^2}{2\pi} \frac{b}{R} \tag{5}$$

S, ratio of outer radius to inner radius (b)

$Z_0 = 377$ ohm

This is the impedance per step.

For the ED we could take S = 1.2 and 2000 steps, then

$|Z/n| = 0.2$ ohm

In the large frequency range the impedance is constant

$$|Z| = \frac{Z_0}{4} (S-1) \tag{6}$$

For the ED with the usual number this gives a total contribution to Z_R of

$$Z_R = 38 \, K\Omega$$

Observe that in both (5) and (6) the real part and imaginary part are equal. The contribution to Z_n in the large frequency range of the plates and bellows will be estimated in a successive note.

Also the impedance can be estimated indirectly from the beam behavior observations[6,25-27]. Some data are shown in the following table. One more remark: the contributions of items to the impedance Z_n could be divided into two groups: the first group of contributions depends on the machine circumference (bellows, pickups,...),

<div align="center">Table I. Some Impedance Values</div>

machine	$|Z/n|$	Z_R
ISR[25]	25 ohm	-
Main Ring	10-50	(100 $\kappa\Omega$)*
SPEAR[6]	$\approx$10 ohm	10 $\kappa\Omega$
PEP	few ohm[(**)]	-

*This number is required to fit some computer simulation to the Main Ring; it also fits an observed alleged momentum variation at flattop at 100 GeV.

**Estimated and required.

the second group is made of lumped elements like RF cavities and its contribution does not change much with the size of the machine.

In the following, to some extent arbitrarily, we shall assume for the Energy Doubler

$$|Z_n/n| \sim 50 \; \Omega \qquad \text{and} \qquad Z_R = 100 \; \kappa\Omega.$$

4. Longitudinal Beam Emittance in the Main Ring and Energy Doubler

A crucial parameter to investigate the stability of a

beam is the longitudinal phase space area of a bunch. Unfortunately this is quite an unknown especially because it does depend on the stability itself.

We shall give here a few definitions:

S = bunch area in eV·s, assuming bi-gaussian distribution and including 95% of the bunch, then

$$S = 6\pi \sigma \delta E / c \tag{7}$$

σ, rms bunch length

δ, rms relative energy spread

E, total energy of a particle.

In the Main Ring and Energy Doubler one has for buckets not completely full

$$\sigma = 8.2 \sqrt{S / \sqrt{EhV}} \quad m \tag{8}$$

$$\delta = 1.94 \times 10^{-3} \sqrt{S \sqrt{hV/E^3}} \tag{9}$$

where V is the total RF voltage in MV, E is in GeV, and h is the harmonic number.

Another relation that we need for the following is

$$\frac{\Delta p}{p}, \text{ relative momentum } FWHH = 2.355 \, \delta \tag{10}$$

Similarly we can also derive a relation between the peak current I_p within a bunch and the average current per bunch I_b

$$I_p = \frac{R\sqrt{2\pi}}{\sigma} I_b \tag{11}$$

5. Longitudinal Stability of Coasting Beam[4,19,28,29]

In some of the colliding-beam schemes, especially for p-p̄, it is required to debunch the beam at either 80 or 100 GeV.

It is therefore necessary to investigate the beam stability for this operation.

The theory predicts the following stability condition

$$\left| Z/n \right| \lesssim \frac{|\eta| E}{e I_o} \left(\frac{\Delta p}{p} \right)^2 \tag{12}$$

where

I_o, average beam current

$\eta = \gamma_t^{-2} - \gamma^{-2} \sim 0.0028$ for $\gamma \gg \gamma_t$.

The growth rate in absence of Landau damping at the onset of the instability is

$$1/\tau = n \omega_o \sqrt{\frac{e I_o |Z/n| |\eta|}{2 \pi E}} \tag{13}$$

If we take

E = 100 GeV

I_o = 0.15 A $(2 \times 10^{13}$ ppp)

$\left| Z/n \right|$ = 50 ohm

then

$1/\tau \lesssim 0.05$ n sec^{-1}

and

$$\left(\frac{\Delta p}{p} \right)_{threshold} \gtrsim 1.6 \times 10^{-4}$$

The threshold value of the momentum spread corresponds to the following bunch area

$$S_{th} \gtrsim 0.5 \ eV \cdot s$$

The growth time of the instability is already 20 msec around the RF frequency and only 1 msec, dangerously too fast, at 1 GHz.

If the debunched beam is held at the threshold of stability

it certainly can be kept safely within the momentum aperture of
the Main Ring or of the Energy Doubler. The subsequent rebunching,
though, at a lower harmonic number could lead to a too large area
and momentum spread to be handled safely.

6. Microwave Instability

This is an instability that develops around the contours
of the bunch in a similar fashion to the coasting beam. So far
three different models have been provided.

a. Coasting Beam Theory Applied to Bunches.[26,30] In the
case the wavelength $\lambda = 2\pi R/n$ of the instability is considerably
smaller than the bunch length and the growth time is also con-
siderably smaller than one phase oscillation period, the main
portion of the bunch can be regarded as a chopped coasting beam
and the Equations (12) and (13) would apply, provided that the
average current is replaced with the peak current according to (11)
and one takes the central momentum spread. The calculations are
done by also combining (12) and (13) with (7), (8), (9), (10)
and (11). The results for E = 1000 GeV, which corresponds to the
worst case, are shown in Table II. We show data for two different
values of Z/n and for two RF voltages V as well as for two dif-
ferent numbers N_B of particles per bunch. Observe that:

> (i) The instability growth time is rather small,
> certainly smaller than one phase oscillation
> period, for frequencies larger than few
> 100 MHz. Therefore the coasting beam theory
> applies rather well.

> (ii) Large currents correspond to large bunch
> spreads. Yet even for $N_B = 10^{11}$ the beam
> size seems to be quite manageable if it is

Table II. Bunch Paramteres at the Threshold
According to the Coasting Beam Theory

$|Z/n|$ = 50 ohm

	V = 1 MV		V = 4 MV	
	$N_B = 2 \times 10^{10}$	$N_B = 10^{11}$	$N_B = 2 \times 10^{10}$	$N_B = 10^{11}$
σ	27 cm	47 cm	17 cm	29 cm
δ	0.6×10^{-4}	1.1×10^{-4}	0.8×10^{-4}	1.4×10^{-4}
S	1.2 eV·s	3.4 eV·s	0.9 eV·s	2.7 eV·s
I_p	1.4 A	4.2 A	2.3 A	6.6 A
$n\tau$	18.9 s	10.9 s	14.7 s	8.7 s

$|Z/n|$ = 10 ohm

	V = 1 MV		V = 4 MV	
	$N_B = 2 \times 10^{10}$	$N_B = 10^{11}$	$N_B = 2 \times 10^{10}$	$N_B = 10^{11}$
σ	16 cm	27 cm	10 cm	17 cm
δ	0.35×10^{-4}	0.6×10^{-4}	0.5×10^{-4}	0.8×10^{-4}
S	0.4 eV·s	1.2 eV·s	0.3 eV·s	0.9 eV·s
I_p	2.4 A	7.2 A	3.8 A	11.3 A
$n\tau$	24.7 s	14.3 s	19.7 s	11.4 s

kept at the threshold of stability.

b. Resistive Theory[10,11,32]. In this model a resistive impedance Z_R is taken constant over the entire frequency range. This impedance would then cause first a local energy loss and then dipole (and eventually higher order) oscillations. After smearing of the coherent oscillations the bunch has reached a new larger phase space area. This model can be easily tested on computer.[11]

To fit the scarse experimental evidences on the Main Ring[32] an impedance of 100 $\kappa\Omega$ is at least required. From the computer simulations one derives the following stability criterion

$$S \gtrsim 0.53 \; \frac{I_b \, Z_R}{\omega_o} \, \gamma_t \sqrt{\frac{E}{eV}} \qquad , \quad (\gamma \gg \gamma_t) \qquad (14)$$

The results for the Energy Doubler at 1000 GeV with Z_R = 100 $\kappa\Omega$ are shown in the following table.

Table III.   Bunch Parameters at the Threshold
According to the Resistive Theory

	V = 1 MV		V = 4 MV	
	$N_B = 2 \times 10^{10}$	$N_B = 10^{11}$	$N_B = 2 \times 10^{10}$	$N_B = 10^{11}$
S	0.5 eV·s	2.5 eV·s	0.25 eV·s	1.3 eV·s
σ	18 cm	39 cm	9 cm	20 cm
δ	4.4×10^{-5}	9.8×10^{-5}	4.4×10^{-5}	9.8×10^{-5}
I_p	2.1 A	4.9 A	4.3 A	9.6 A

The beam parameters are not much different from those shown in Table II which indicates an equivalence between the two "fitting" parameters $|Z/n|$ and Z_R.

The bunch area at the threshold is not really very large, it can be easily accommodated in an RF bucket of 1 MV and stationary. At 1000 GeV the bucket area would be 10 eV·s. Also a momentum aperture of 10^{-3} (total) would be required in the Energy Doubler.

c. Low-Q Resonator Model[8,9]. In this model the total impedance Z of the machine is approximated to that of a low-Q resonating circuit. Three parameters are required: the frequency of resonance (f_0), the shunt impedance Z_R and the inductance (Z/n). This model is based on some phenomenological considerations (see section 2 of this paper) and on some experimental evidences in SPEAR[6]. Also this model as the previous one is suited for numerical simulations.

The result is again a stability criterion of the form (12) where I_0 is still the average current (total) in the beam of n_B bunches, n is now the ratio of the resonance frequency F_0 to the revolution frequency $\omega_0/2\pi$ and

$$Z = \frac{1}{2\pi} \sum_m Z_m \, e^{-(m-n)^2 \frac{\sigma^2 n_B^2}{R^2}} \tag{15}$$

This quantity therefore depends on the bunch rms length σ. Also $\Delta p/p$ should be replaced by the rms energy spread δ at the r.h. side of (12).

In the limit of a broad-band resonator (15) becomes

$$Z = \frac{R Z_R}{\sqrt{4\pi} \, \sigma \, n_B} \tag{16}$$

Combining (12) and (16) gives ($\gamma \gg \gamma_t$)

$$\delta^2 \sigma \gtrsim \frac{R Z_R I_b \gamma_t^2}{\sqrt{4\pi} \, n \, E/e} \tag{17}$$

where I_b is the average current per bunch. The threshold bunch area (in eV·s) is given in Table IV for different Z_R and N_B (particles per bunch). Though there is quite a different parametric difference between (17) and (14), there is nevertheless not much quantitative difference.

Table IV. Threshold Bunch Area (eV·s) According
to the Low-Q Resonator Model

V = 1 MV,   n = 2×10^4 ($f_o \sim 1$ GHz)
E = 1000 GeV,   h = 1113

N_B \ Z_R	0.1 MΩ	1.0 MΩ
2×10^{10}	0.18	0.82
10^{11}	0.52	2.40

7. Overshoot

In the previous sections we have estimated the bunch size at the threshold of the stability. But what happens when the beam is below the threshold, namely the stability conditions (12) and (14) are not satisfied? The answer is given by several computer simulations. The first computer exercise was performed by Dory[31] more than sixteen years ago. His main result is shown in Fig. 2, and applies to coasting beams. The Dory's overshoot formula is

$$\left(\frac{\Delta p}{p}\right)_{final} \cdot \left(\frac{\Delta p}{p}\right)_{initial} = \left(\frac{\Delta p}{p}\right)^2_{th} \tag{18}$$
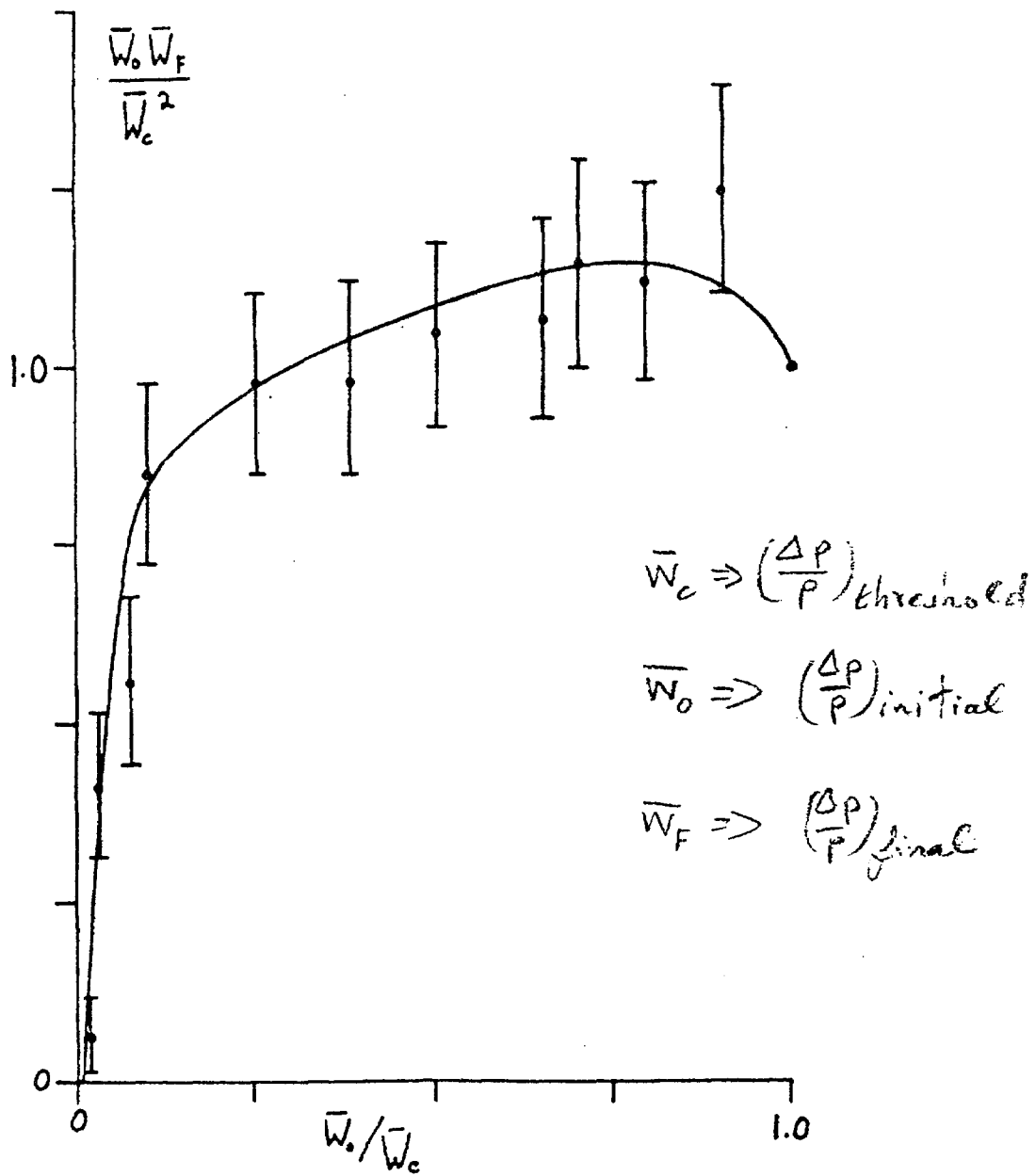
Figure 2.   Dory's Overshoot Result

which relates the final momentum spread to the initial value
and the threshold as given by Eq. (12). According to Dory's
result an unstable beam will initially grow its momentum spread,
but the growth will stop once a final value is reached which
satisfies Eq. (18). Therefore, provided there is enough aperture,
an instability not necessarily would cause a beam loss but only
a beam enlargement. It seems therefore, that once an instability
is acknowledged it is convenient to maintain the beam continuously
at the threshold to minimize the beam growth.

The overshoot formula (18) was later substantiated with
more computer studies for coasting beams[7] and also for bunched
beams[8,9,11]. Actually for bunched beams the overshoot formula
can also take the form[32]

$$S_{final} \cdot S_{initial} = S_{th}^{2} \tag{19}$$

which involves the bunch area rather than the momentum spread.
At the r.h. side of (19) one can use, for instance, the threshold
(14).

The overshoot equations are now quite usually employed to
represent the behavior of <u>proton</u> beams.

8. Possible Cures

As we have seen the size of the beam at the threshold of
stability is small enough in any case investigated to be easily
kept safely within the momentum aperture of the Energy Doubler.
Therefore it seems quite crucial to maintain the beam always,
constantly at the limit of instability. This not only would
eliminate an overshoot behavior which would lead to larger beam size,
but also would keep the beam as even as possible with no internal
oscillations.

The beam can be kept at the limit of stability with a "Bunch Spreader" which continuously should blow up the bunch the minimum amount required during the entire cycle. Therefore the installation of such as device which we call here Bunch Spreader is quite crucial and should be designed very carefully and with adequate dynamical range.

If the smaller beam sizes are required other cures should be provided to overcome the instability. There are three possibilities to our knowledge:

(i) "Smooth up" the vacuum chamber. Namely reduce $|Z/n|$ and Z_R. This can be accomplished by keeping an inventory of items that are installed in the Energy Doubler vacuum chamber, and maintain a watch on their effect by calculation or/and measurements.

(ii) Fast longitudinal damper for individual bunch oscillations. This damper though can compensate only for dipole oscillations and may be for quadrupole (bunch shape) oscillations. Higher order modes are difficult to be detected because of the relative short length of the bunches. The bandwidth of the system depends on the number of bunches, it can range from 1 MHz for 10 bunches to about 100 MHz for 1000 bunches. The amount of gain required (to be estimated) depends on the growth time of the instability.

(iii) A high harmonic cavity (like the one of the CEA kind presently installed in the Main Ring) could be used for a dynamical compensation of the energy loss induced by the beam according to the resistive theory.

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